

# Minimize the Noise Error Using Simulation of Discrete Kalman Filter

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**ABSTRACT:** The linear Kalman–Bucy filter has seen considerable application to engineering problems involving both linear and nonlinear systems. The generally successful application of this procedure has been marred by the not uncommon appearance of the so-called divergence phenomenon. In our paper the “Kalman filter” is used as recursive estimation which is sufficient important to minimize the errors in discrete system by using matlab simulation.

## 1 INTRODUCTION

In Kalman filter and Bucy, the state and parameter estimation problem initially without reference to the control problem, the inclusion of control inputs is trivial [1].

In 1960 Kalman and Bucy 1961 employed by control engineers and other physical scientists has been successfully used in such diverse areas as processing of signals in aerospace tracking and underwater sonar and the statistical control of quality [1].

Many practitioners of statistics are not aware of the simplicity of this useful methodology. However, the model, the notations, and the techniques of Kalman filtering are potentially of great interest to statisticians doing to their similarity to linear models of regression and time series analysis, because of their great utility in applications [2].

## 2 FILTER THEORY:

Divergence is caused by errors in the model assumed for the filter; most commonly, errors in the model of the plant constitute the dominate source of difficulty. Since the filter requires a linear model, errors can result either from the basic description of the system

(possibly nonlinear) or from the approximations required obtain a linear system. The filter uses the plant model to relate data obtained at sampling times  $t_0, t_1, \dots, t_{k-1}$  to the state at the "current" time  $t_k$ . As the time interval increases, the model errors generally become larger there by destroying the validity of these older data as a source of information about the current state [2].

Divergence is said occur when the actual error in the estimate of the state becomes in consisting with the error covariance predicated of the state becomes in consisting with the error covariance predicated by the filter equations and essentially represents a breakdown in the data processing method [2].

The Kalman filter is a tool that can estimate the variables of a wide range of processes. In mathematical terms we would say that a Kalman filter estimates the states of a linear system. The Kalman filter not only works well in practice, but it is theoretically attractive because it can be shown that of all possible filters, it is the one that minimizes the variance of the estimation error. Kalman filters are often implemented in embedded control systems because in order to control a process, you first need an accurate estimate of the process variables [3].

Many methods have been devised to combat the divergence problem in Kalman filter

applications and each can be considered as a means of diminishing or eliminating the influence of the past data on the estimate of the current state. While this goal is sometimes achieved in rather indirect ways. The basic idea is not new but it does not appear which received the attention that would appear to be warranted in the recursive filtering context. It was introduced in this context by "Fagin" for discrete systems with no plant noise and is generalized here to include both time-continuous and time-discrete systems with plant noise. Fagin refined it as a method for the exponential age. Weighting of old data. More recently, "Morrison" designated the procedure as "fading memory filtering" [2].

### **3 FADING MEMORY FILTERING:**

Fading memory filtering is an outgrowth of considerations relating to deterministic least-squares. In this problem, discounting of past is accomplished through the choice of the least-squares weighting matrices. If all data are treated equally, then the weighting matrices are all the same. If the data obtained at earlier times are to have a smaller influence than more recent data, then these data are discounted by assigning smaller values to the associated weighting matrices. One can show that the Kalman filter equations can be obtained as a solution to the unbiased, linear, mean square filtering problem or as the solution of a deterministic least-squares problem. The two solutions yield the result that the least-square weighting matrices play the same role as the prior covariance matrices of the noise processes of the filtering problem [4].

This approach is often taken the fading memory filter equations for time-continuous and time-discrete systems. The behavior of the error covariance matrix of the resulting filter. Also the fading memory filter is applied in an adaptive manner to a scalar system in order to emphasize the character of the resulting filtering procedure. Fading memory filtering seems to be the most successful and popular way to control divergence effects [4].

### **4 FADING MEMORY FILTERING:**

For this discussion it will be assumed that the basic system can be desired approximately by the following system. The n-dimensional state is represented for some finite interval by [5]:

$$\dot{x}(t) = f(t)x(t) + w(t) \dots\dots\dots (1)$$

And is observed imperfectly at each time through measurement quantities

$$z(t) = h(t)x(t) + v(t) \dots\dots\dots (2)$$

The initial state  $x(t_0)$  is assumed to have value  $a$  and covariance matrix  $M_0$ . The plant and measurement noise process,  $w(t)$  and  $v(t)$ , are zero mean white noise with covariance matrices. The noise processes are assumed to be mutually independent and independent of  $x(t_0)$  for all  $t$ .

It is frequently true that the behavior of the state can be represented adequately by eq.1 over some finite interval of time. However, the use of eq.(1) for more extending periods cannot be adjusted with the consequence in many filtering problems that the resulting model errors cause the occurrence of divergence. the filter bases its estimates  $\hat{x}(t/t)$  on all data.

$Z(s)$ ,  $t_0 \leq s \leq 1$  and is being missed because the early data is no longer accurately related to  $x(t)$  in the manner implied by eq. (1) and eq.(2). Thus, divergence control is essentially the problem of reducing the influence of this data on the determination of  $\hat{x}(t/t)$ . Since the errors resulting from the use of eq.1 would frequently be expected to accumulate gradually rather than to manifest themselves suddenly, the data should themselves be discount gradually.

### **5 THE BASIC FILTER EQUATIONS**

The equations connected with Kalman filtering are collected in Table I [6]:

TABLE I

### MEMORY FILTERING:

As the already been mentioned, the discounting of data can be accomplished

Continue

<i>Kalman filter gain</i>		<i>Conveyances</i>	
$K(k+1) = P(k+1/k)HT(k+1)[H(k+1)P(k+1/k)HT(k+1) + R(k+1)]^{-1}$		$P(k+1/k) = F(k)P(k/k)FT(k) + Q(k), P(0/0) = P0$ $P(k+1/k+1) = [1 - k(k+1)H(k+1)]P(k+1/k)$	
<i>Kalman filter Equations</i>			
<i>System model</i>		<i>Predictor</i>	<i>Corrector</i>
$x(k+1) = F(k)x(k) + w(k)$ $z(k+1) = H(k+1)x(k+1) + v(k+1)$		$x(k+1/k) = F(k)x(k/k), x(0/0) = 0$ $z(h+1/k) = H(k+1)x(k+1/k)$	$x(k+1/k+1) = x(k+1/k)$ $\Delta z(k+1/k) = z(k+1) - z(k+1)$

The Kalman filter for an  $n^{\text{th}}$  – order linear discrete –time stochastic system in another  $n^{\text{th}}$ -order linear discrete –time system. The filter input is the system measurement, and the filter output is the optimally estimated system state, as indicated in Figure 1.



Fig.1 Kalman Filtering a System Output to Optimally Estimate the System State.[2]

Figure 2 shows a block diagram showing the relationships between the signals in the Kalman filter. The filter consists of a model of the system with zero noise inputs replacing the actual (unknown) system noise inputs and initial conditions determined by the measurement residuals through the Kalman gain K.

through the choice of the covariance matrices. With this in mind define a filter model in the following manner. At the current time , the actual and data be given by:

$$z(t) = h(t)x_T(t) + v_T(t) \dots \dots \dots (3)$$

Note that  $z(t)$  appears on the left-hand side of both eq's. (1-2) and eq's.(1-3) . This is done to emphasize that the data  $z(t)$  in a given application are known realizations so are not affected by a change in the model. However, to compensate for model in accuracies that result from the continued utilization of eq.(1) and eq.( 2) , the fading memory filter assumes the data are more adequately represented by eq.(3) in which the measurement noise  $v(t)$  is replaced by noise  $v_T(t)$  with a larger covariance [6].

### 7 DISCRETE- TIME FADING MEMORY FILTERING:

The results of the preceding section can be extended without difficulty to time- discrete systems. That is, suppose the state behavior is described by a linear difference equation instead of eq.(1) and that the measurements eq.(2) occur only at discrete instants:

$$X_k = \Phi_{k,k-1}X_{k-1} + W_{k-1} \dots \dots \dots (4)$$

### 6 CONTINUOUS – TIME FADING

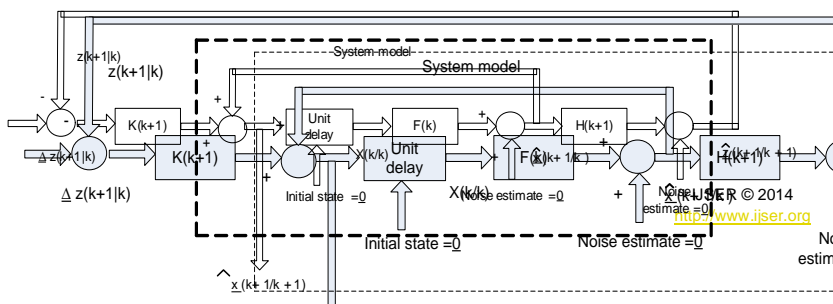


Fig2. Block Diagram of the Kalman Filter.[2]

$$Z_k = H_k X_k + V_k \quad \dots\dots\dots (5)$$

Suppose that the filter model is given by:

$$X_k^N = \Phi_{k,k+1}^N + W_{k-1} \quad \dots\dots\dots (6)$$

$$Z_k = H_k + X_k^N + V_k^N \quad \dots\dots\dots (7)$$

Where the superscript N is viewed as current time.

## **8 STABILITY OF FADING MEMORY FILTER:**

Figure 1 illustrates block diagram design description of a second order Fading Memory Filter (FMF).

The design and operation of this filter can be described by the following recurrence relations:

$$X_n = X_{n-1}^0 + X_{n-1}^0 T_s + G[X_n^0 - (X_{n-1}^0 + X_{n-1}^0 T_s)] \quad \dots\dots\dots (8)$$

$$X_n^0 = X_{n-1}^0 + H/T_s [X_{n-1}^0 - X_{n-1}^0 + X_{n-1}^0 T_s] \quad \dots\dots\dots (9)$$

Where x is the input sinusoidal signal,  $x^0$  is the distance which represents one of FMF outputs,  $x_n^0$  is the first order derivative of  $x^0$  which represents the velocity,  $T_s$  is the time delay, and G is the gain of the filter. The FMF gain is given by

$$G = 1 - \beta^2 \quad \dots\dots\dots (10)$$

where  $\beta$  has values  $0 < \beta < 1$ .

$$H = (1 - \beta)^2 \quad \dots\dots\dots (11)$$

The Fading Memory Filter can be used to reduce the noise associated with the signal transmission, in addition the distance, velocity and the acceleration can be generated without need for measurements, and also, the gain is

constant in the FMF, so there is not need for differential equation to calculate the gain .

## **9 KALMAN FILTER MODEL (FMF SIMULATOR DESIGN AND OPERATION):**

Figure 3 depicts the detailed block diagram description of the Matlab simulate the Fading Memory Filter.

The simulator first generates a sinusoidal wave, and a noise signal using the random number generator, then adds the input sin and the noise. The resultant signal is then processed by the Zero-Order Hold circuit and displayed on the scope (scope3).

After the required multiplications by the gain factors being applied, and the unit delay and all feedback processes being applied, then the first FMF output, i.e. distance X, is obtained, and displayed on (scope4).

On the other hand, the required processing is applied to obtain the second output, this is the velocity. The velocity is displayed on the (scope 5).

All the outputs simulation is shown in figures (4, 5, 6, 7 and 8).

## **10 CONCLUSIONS:**

There are several sources of good up-to-date software specifically designed to address the numerical stability issues in kalman filtering. Scientific software libraries and workstation environments for the design of control and signal processing systems typically use the more robust implementation methods available.

In our situations where noise contaminated measurement must be used (for example radio or radar receiver ), the Kalman-Bucy filter which used to reconstruction the state vector from noisy output signals, then the outputs of scope 1 in goes with scope 2 (fitted) after for 10sec. as window.

Although, its use our results in optimal rejection of the noise signals which corrupt the measurement, it requires a dynamics system of the same order at any dynamic system.

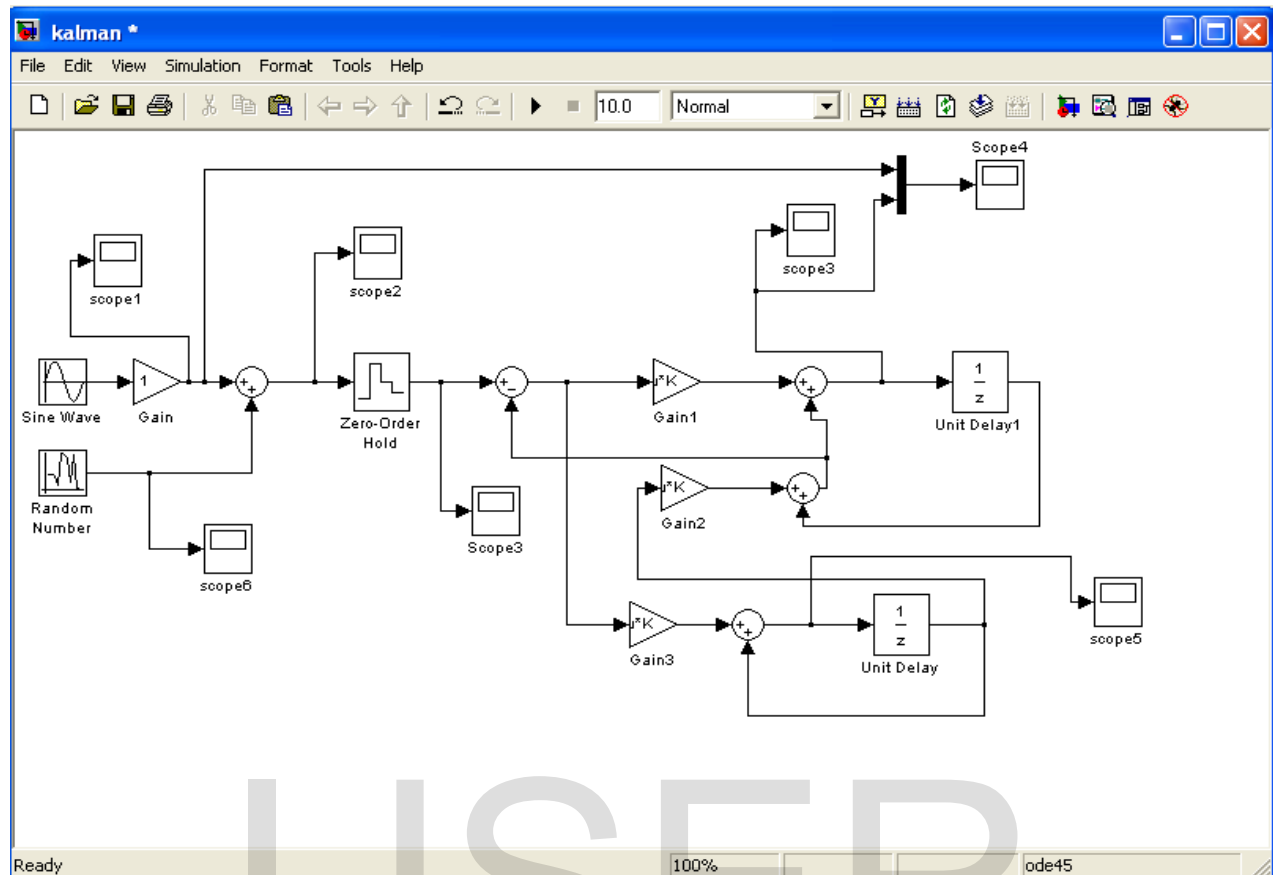


Fig 3. Simulation Block Diagram.

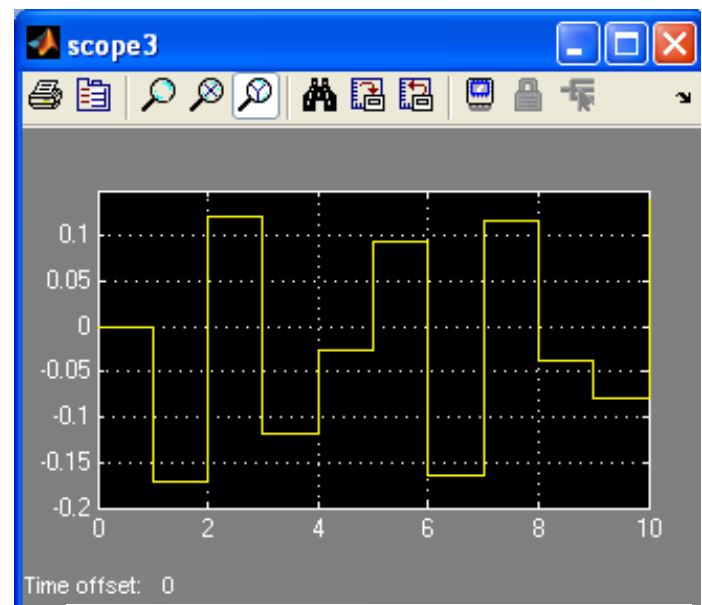
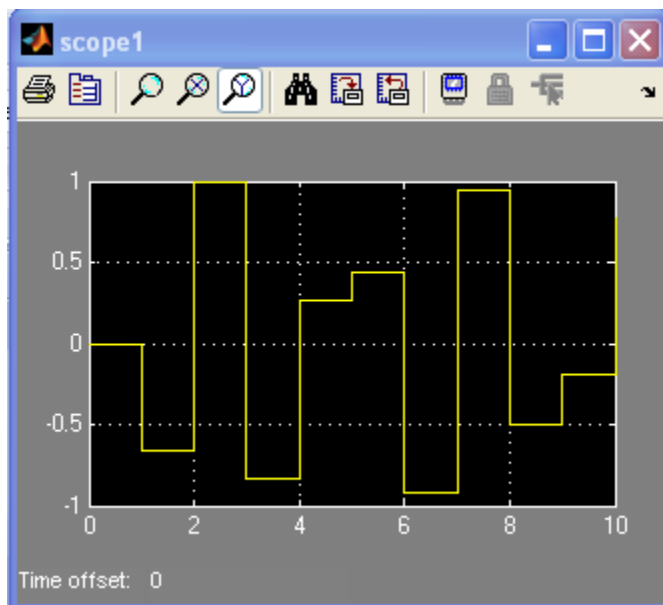


Fig 4 : the output of scope 1

Fig 5 : the output of scope 3

Fig.7: The Otput of Scope 5.

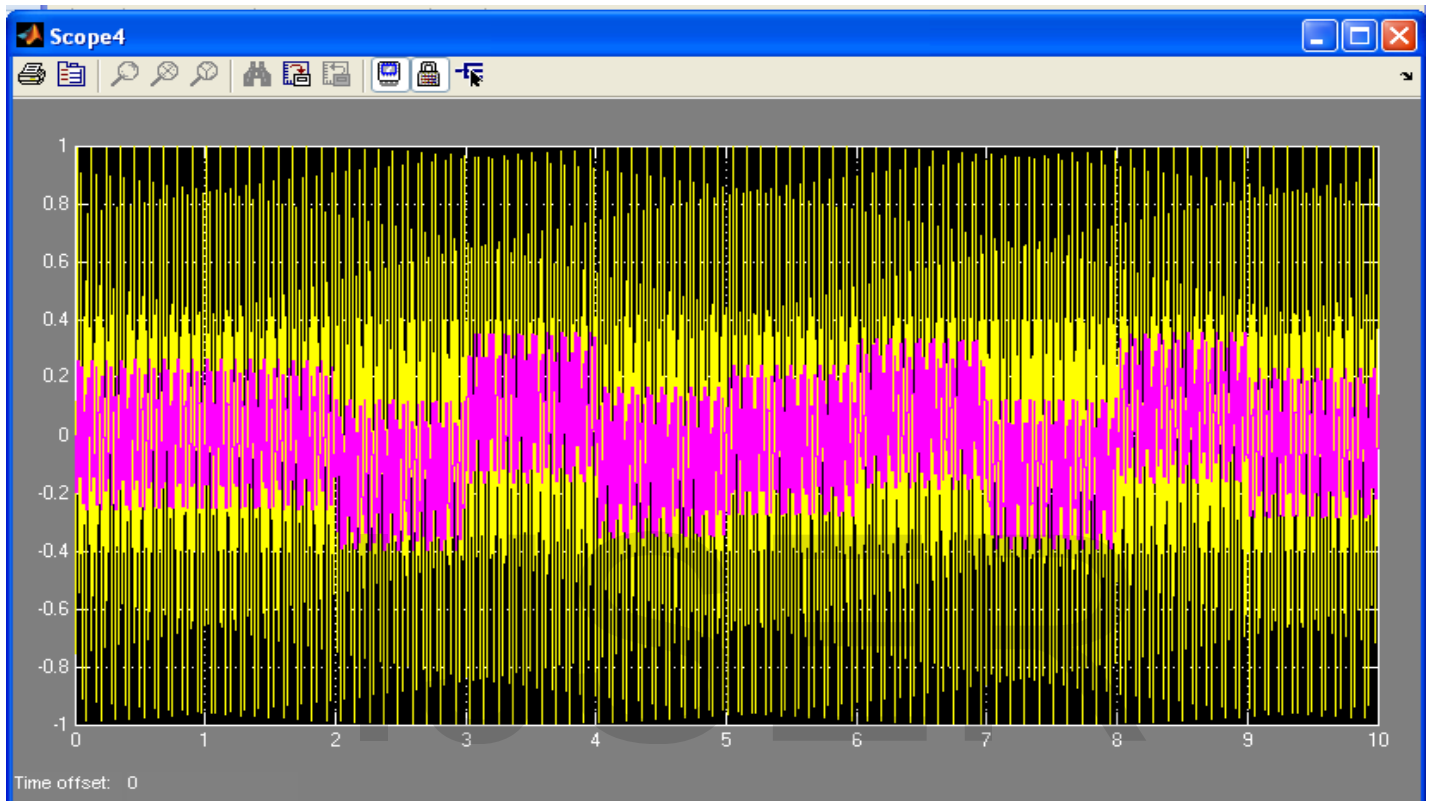
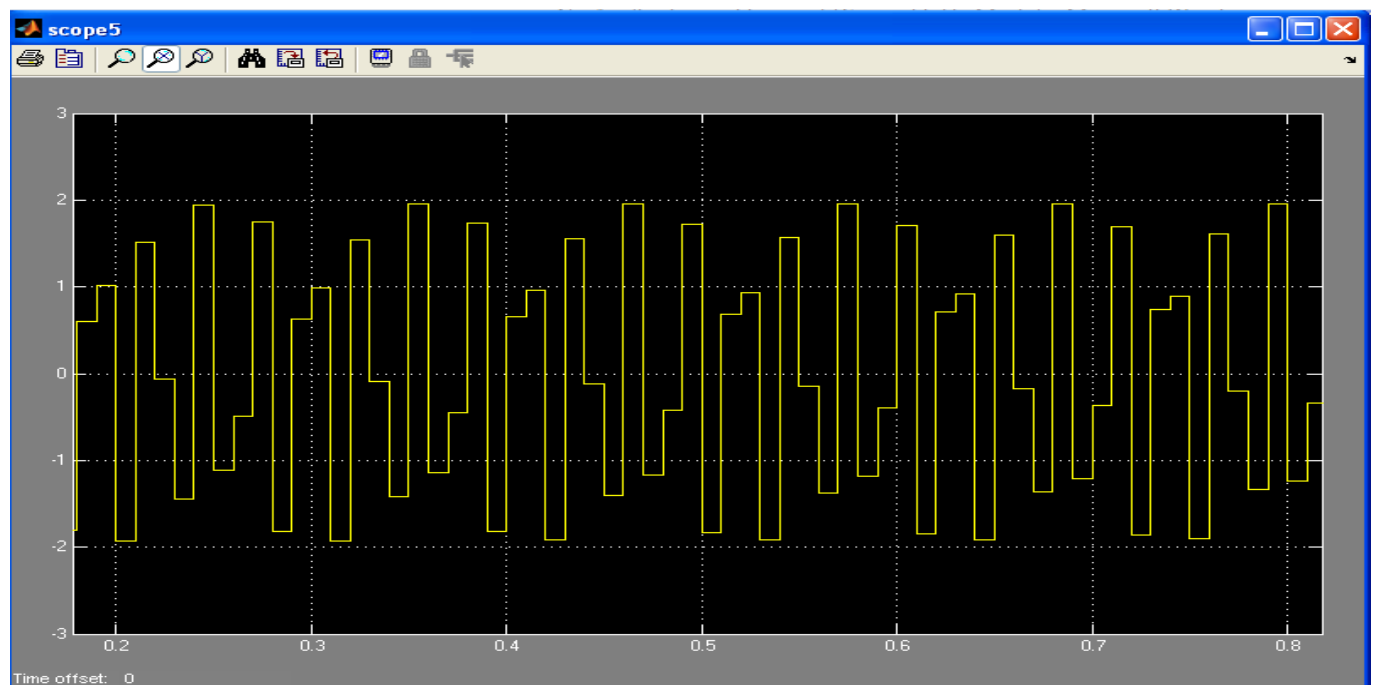


Fig6: The output of scope 4



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